Test 3 Mechanics & Relativity 2018-2019

Tuesday October 30, 2018, 9:00 – 11:00, Aletta Jacobshal

Before you start, read the following:
There are 4 problems for a total of XX points
Write your name and student number on each sheet of paper
Do no separate the exam-strack and try to fit all answers on them
Spare sheets are added at the back of the stack
Make clear arguments and derivations and use correct notation
Support your arguments by clear drawings where appropriate
Write in a readable manner, illegible handwriting will not be graded

Three-vectors are printed italic + bold-face: $\boldsymbol{p}$
Four-vector are printed italic: p
Length of a vector is indicated by |...|: $|\boldsymbol{p}|$

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STUDENT NUMBER: s...

Problem 1	**: .... points out of 10**
Problem 2	**: .... points out of 15**
Problem 3	**: .... points out of 15**
Problem 4	**: .... points out of 14**
Total	**: .... points out of 54**

GRADE = 1 + #points/6 =

Lorentz Transformation equations, with

$$\begin{aligned} t' &= \gamma(t-\beta x)\ , \qquad t = \gamma(t'+\beta x')\ , \\ x' &= \gamma(x-\beta t)\ , \qquad x = \gamma(x'+\beta t')\ , \\ y' &= y\ , \\ z' &= z\ . \end{aligned}$$

Einstein velocity transformations

$$\begin{aligned} v'_x &= \frac{v_x-\beta}{1-\beta v_x}\ , \qquad v'_y = \frac{v_y\sqrt{1-\beta^2}}{1-\beta v_x}\ , \qquad v'_z = \frac{v_z\sqrt{1-\beta^2}}{1-\beta v_x}\ , \\ v_x &= \frac{v'_x+\beta}{1+\beta v'_x}\ , \qquad v_y = \frac{v'_y\sqrt{1-\beta^2}}{1+\beta v'_x}\ , \qquad v_z = \frac{v'_z\sqrt{1-\beta^2}}{1+\beta v'_x}\ . \end{aligned}$$

NAME: ....

STUDENT NUMBER: s...

Problem 1 – Basics (10 points)

Indicate whether a statement is TRUE (T) or FALSE (F)
by placing an ✕ in the corresponding box: ☒.
You can make a correction by completely blacking out the wrong answer: ■
Score = #correct answers – 10 (minimum 0)

a) Albert Einstein was the first to formulate the principle of relativity — T: ☐ F: ☒

b) The ratio $|\mathbf{p}|/E$ is frame-independent — T: ☐ F: ☒

c) If m=0, the ratio $|\mathbf{p}|/E$ is frame-independent — T: ☒ F: ☐

d) Four-momentum is frame-dependent — T: ☐ F: ☒

e) Two events connected via a space-like interval cannot be causally connected — T: ☒ F: ☐

f) The time-components of the four-momentum is defined as $E = \mathrm{d}\tau/\mathrm{d}t$ — T: ☐ F: ☒

g) $\beta^2 + 1/\gamma^2 = 1$ — T: ☒ F: ☐

h) Spacetime intervals may be frame-dependent — T: ☐ F: ☒

i) Because light has no mass, its momentum is zero — T: ☐ F: ☒

j) Total four-momentum is conserved in collisions — T: ☒ F: ☐

k) $E^2 + |\mathbf{p}|^2 = m^2$ — T: ☐ F: ☒

l) Each of the components of the total four-momentum is conserved — T: ☒ F: ☐

m) The mass of a system of particles is equal to the sum of the particle masses — T: ☐ F: ☒

n) Relativistic momentum is defined as $\mathbf{p} = m\,\mathbf{v}$ — T: ☐ F: ☒

o) When measured along a straight worldline, $\Delta t = \Delta\tau$ — T: ☐ F: ☒

p) Events outside each other's light cones are causally un-connected — T: ☒ F: ☐

q) Because the Lorentz transformation leaves the components *perpendicular* to the relative motion between two inertial frames unaffected, the corresponding velocity components are also the same in both frames — T: ☐ F: ☒

r) Length contraction is frame-symmetric — T: ☒ F: ☐

s) From the perspective of one reference frame, clocks in another moving reference frame are running slower — T: ☒ F: ☐

t) Events on a line parallel to the x'-axis share the same time t — T: ☐ F: ☒

NAME: ....
STUDENT NUMBER: s...

Problem 2 – Space Tragedy? (15 points)

Two heroic space-explorers decide to travel to a recently discovered earth-like planet. They leave Earth in their spaceship which has a cruising speed of 2/5*c*. After two years (according to the clock in their spaceship) they discover that their air supply system is malfunctioning, and that they have already consumed 50% of the available air. Via their radio set they send an SOS signal back to Earth, with the request to send help. Upon receiving this message, Earth responds by radio that help is underway. They also launch the newest Scintilla-90 (cruising speed 0.9*c*) rapid-response ship with extra air-supplies.

a) In the spacetime diagram on the next page add the world lines and events depicting the story described above. Clearly label all events and worldlines. **(4 points)**

b) *Calculate* how long after departure Earth receives the SOS call *and* how far from Earth the explorers are at that time (from Earth's perspective). Give the full calculation. **(3 points)**

First: time + location SOS
$\Delta T[\text{earth}] = \gamma \Delta T[\text{ship}] = \gamma \cdot 2\text{yrs}$; $\gamma = 1/(1-\beta^2)^{½} = 1/(1-2/5^2)^{½} = 1.09 \rightarrow \Delta T[\text{earth}] = 2.18\text{yr}$
Distance traveled from Earth $D = \Delta T[\text{earth}] \cdot \beta = 0.873\text{yr}$
Then: time of receiving SOS on Earth: $T[SOS] = \Delta T[\text{earth}] + D = \mathbf{3.06yr}$ 1½
Last: Distance from Earth when receiving SOS: $D' = D + \beta \cdot D = \mathbf{1.222yr}$ 1½

• correct formula, but wrong nr: 1pt
• only correct nr: ½ pt

c) *Calculate* the speed of the explorer's ship *from the perspective of the rapid-response ship.* Give the full calculation. **(2 points)**

Use Einstein's formula: $v_x' = (v_x - \beta)/(1 - v_x \cdot \beta) = (2/5 - 0.9)/(1 - 2/5 \cdot 0.9) = \mathbf{-0.781}$
only correct v nr, or only formula: 1pt   wrong sign Δ: 1pt max

d) Use *causality* to find out whether the new air-supplies can lead to the survival of the space explorers. *Motivate* your answer through the space-time diagram.
Will the explorers survive? Yes: □ No :■ **(2 points)**

Light cone does not include receipt of SOS on Earth

e) Right after sending the SOS signal, one of the two explorers suspects help will not come in time and decides to abandon ship so that the other explorer has twice as much air left and can thus survive twice as long. Will the second explorer survive? *Motivate* your answer with a calculation or a drawing in the space-time diagram with a brief explanation. **(4 points)**

From Earth, the space ship is 1.222 yrs away and relative speed is (0.9–2/5)=0.5.
Distance=1.222yr, so to get there requires 1.222yr/0.5=2.444yr after receiving the SOS
New air ~~receives~~ arrives at the spaceship 3.06+2.44=5.50yr after it left Earth=5.5~~4~~yr/γ=5.04yr in the (dilated) spaceship frame. This is too late for two explorers, but still in time for a single one.

Motivation: 3pt
Correct answer [yes] 1pt

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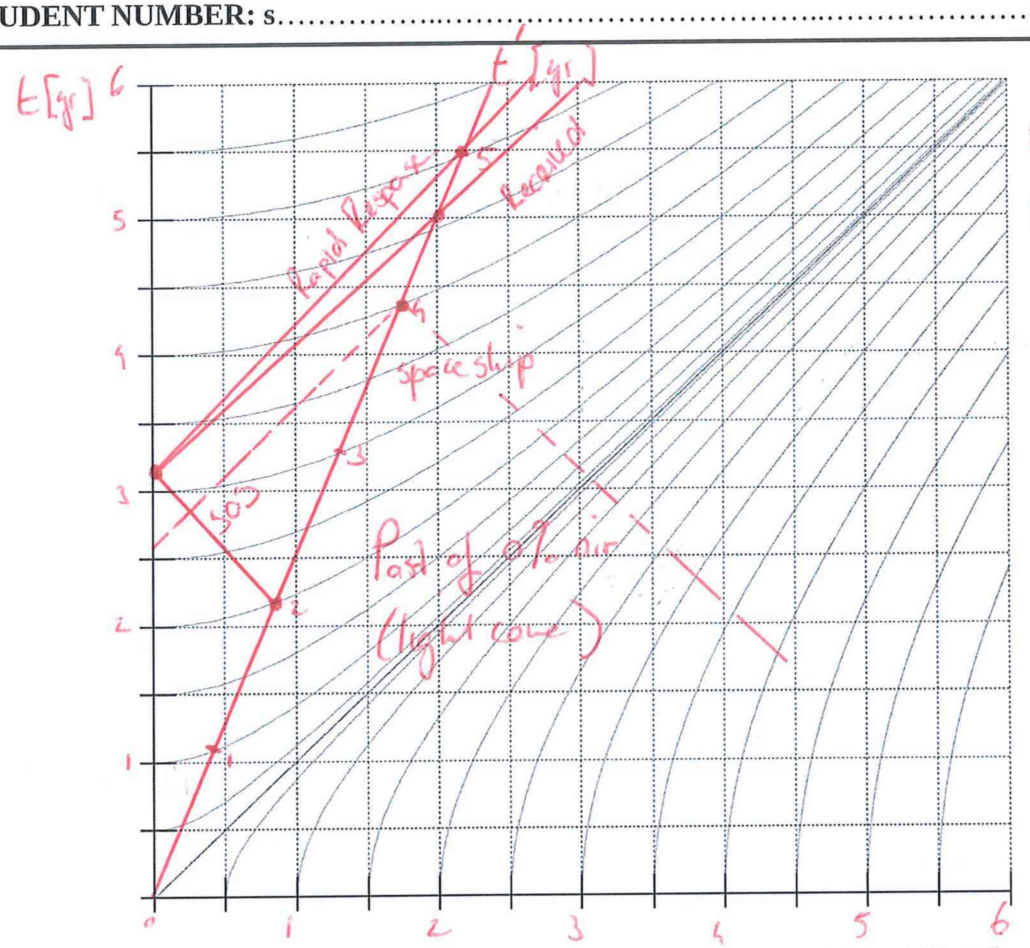
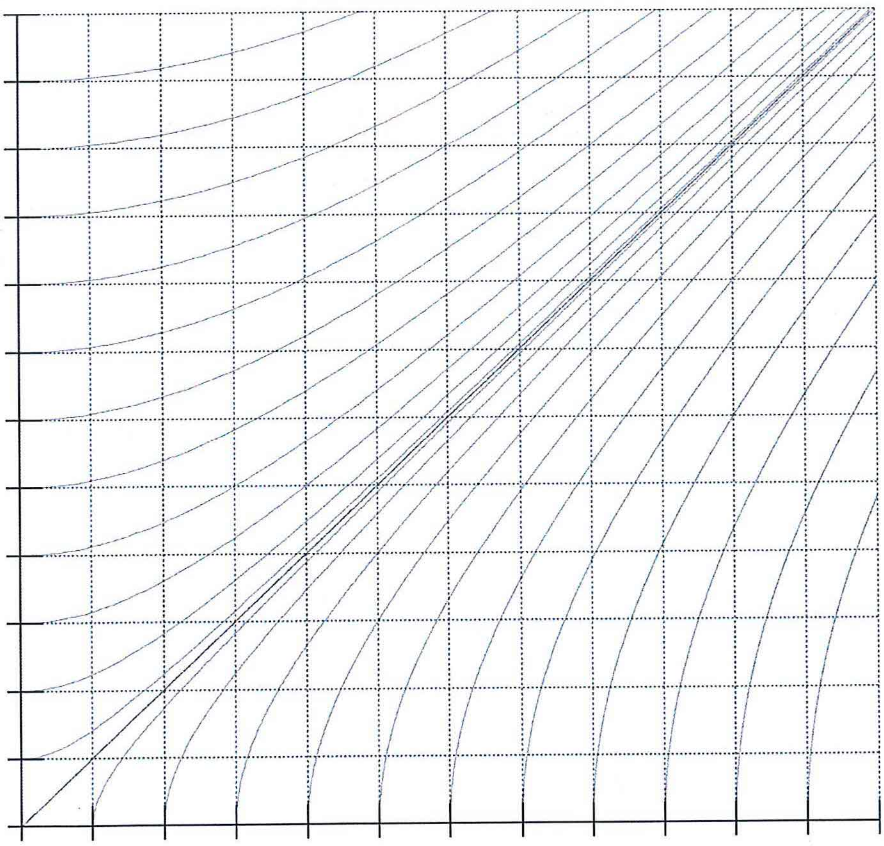
STUDENT NUMBER: s………………………………………………………………………………



- labels : 1
- spaceship @ β=4/5 : 1
- SOS (+ response) @ β=1 : 1
- Rapid response @ β=0.9 : 1

Spare diagram:



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STUDENT NUMBER: s..

Problem 3 – Accelerators (14 points)

Particle accelerators increase the kinetic energy of a particle. Two basic approaches are used to study particle properties. In the first approach **(A)**, one of two particles is accelerated and then it collides with the second one that is at rest. In the second approach **(B)**, both particles are accelerated and then collide head-on.

a) Give the expressions for the total energy E, relativistic momentum $|\boldsymbol{p}|$, speed $|\boldsymbol{v}|$, and four-momentum p of a particle with mass m and kinetic energy K. **(4 points)**

Expressions should only depend on m & K

1 E=m+K
1 |p| = (E² – m²)^½ = [(m+K)² – m²]^½ = [m² + 2mK + K² –m²]^½ =[2mK + K²]^½
1 |v| = |p|/E = [2mK + K²]^½ /(m+K)
1 p = (E, **p**) = (m+K, |p| $\hat{p}$) ← or form with p_x, p_y, p_z or with |p| and angles (no preferred form)

b) Give the expression for the mass of a system of two particles, respectively with masses m_1 and m_2 and relativistic three-momenta $\boldsymbol{p_1}$, and $\boldsymbol{p_2}$. (*only simplify if practical*) **(2 points)**

vector sum : 1 pt

M² = (E1+E2)² – (**p1**+**p2**)² = (√(m1²+p1²)+√(m1²+p1²))² – (**p1**+**p2**)²

1/2 pt. 1/2 pt

c) Two colliding particles can create a single new particle, **1+2→3**. Compare the following two setups: in method **(A)** described above the accelerated particle has kinetic energy *2K*, whereas in method **(B)** both particles have kinetic energy *K*. In the E-p diagram on the next page draw the four-momenta of both particles for both methods (clearly label them), indicate below which of the two approaches, (A) or (B), produces the largest mass for particle 3 and *very briefly* explain why. Use $m_1 = m_2 = 100$ MeV, and $K = 150$ MeV. **(4 points)**

1pt

In both cases the total energy is the same. In case (B) the total momentum is zero, so all energy is available to create mass. In case (A) part of the energy is used for kinetic energy since the total momentum is not zero

d) For both setups described above, calculate m_3 for the case that $m_1 = m_2 = 500$ MeV and $K = 250$ MeV. **(4 points)**

Etot, ptot, M3 — 1/2 pt each
1/2 pt for formula
1/2 pt for number

(A): E1=m1+2K=1000MeV; E2=m2=500MeV; p1=sqrt(E1²–m1²)=866MeV; p2=0
Etot=E1+E2=1500MeV; ptot=p1+p2=866MeV; M3=sqrt(Etot² – ptot²)=1224MeV
(B): E1=E2=m+K=750MeV; p1=p2 (no need to calculate)
Etot=E1+E2=2m+2K=1500MeV; ptot=0; M3=Etot=1500MeV

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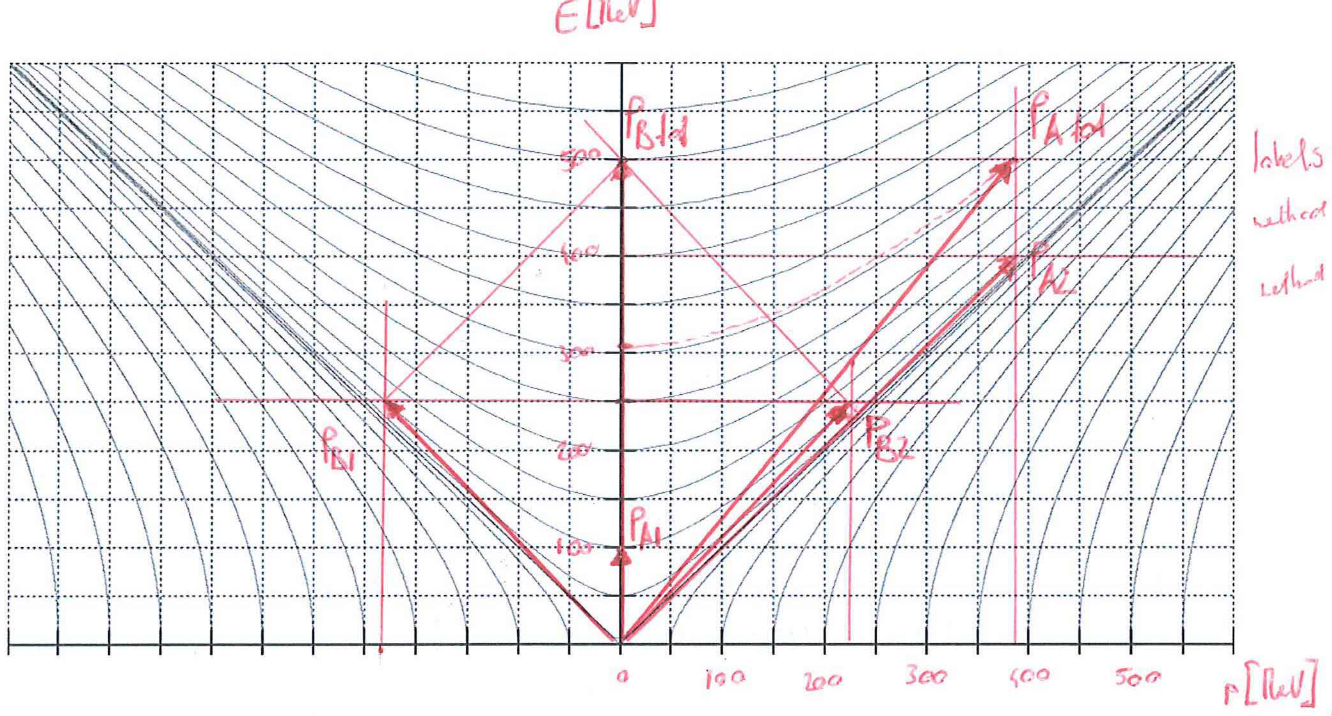
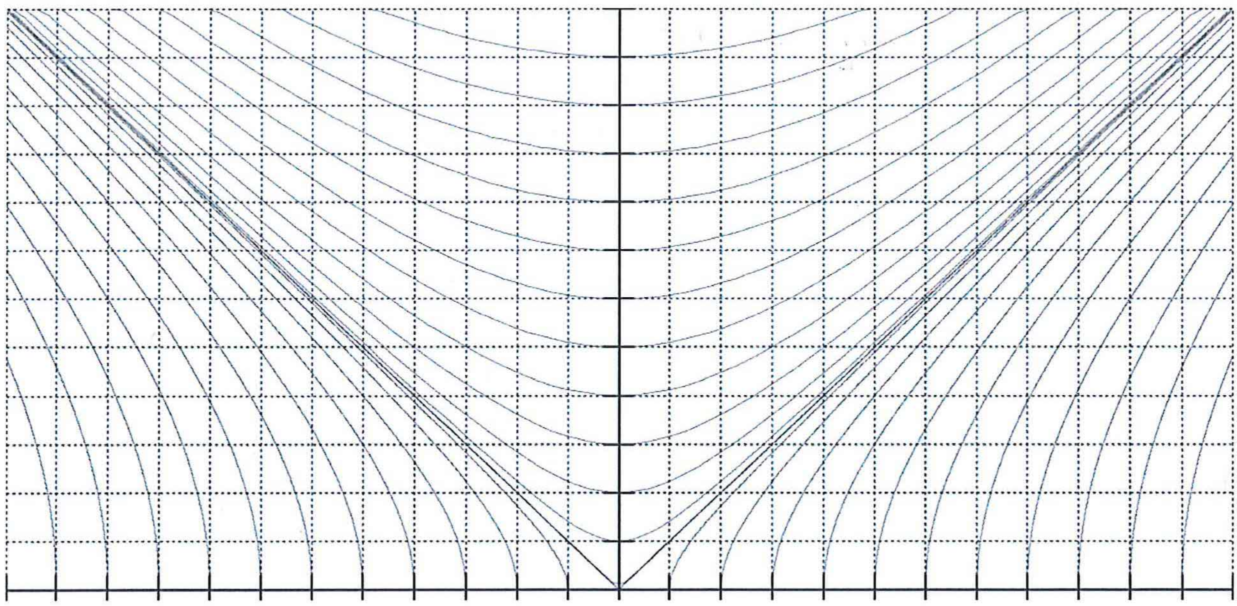
STUDENT NUMBER: s……………………………………………………………………………………………



labels: 1pt
method A: 1pt
method B: 1pt

Spare diagram:



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STUDENT NUMBER: s……………………………………………………………………………

Problem 4 – Encyclopædia Britannica (15 points)

The Encyclopædia Britannica is a general-knowledge English-language encyclopaedia. The printed version requires about 3.5 m (11½ ft) of bookshelf. Guesstimate how many times the letter "e" appears in it.

⅓–1 2
1–3 3
3–10 4
10–100 5
100–300 4
300–1000 3
1000–3000 2
3000–10000 1

Final answer: N[e] = 30M   5 pt – 1 per factor 3

(Briefly) explain the variables used, including their input values (if not calculated from others):
L=3.5m : total length (thickness) of books
d=0.1mm : thickness of a sheet of paper
NP : number of pages (32,640 in reality)
NL = 100 : lines per page
NWL = 15 : words per line
NWP : words per page
NW : total nr words (40M in reality)
NCW = 5: characters per word (4.5 in reality)
NC : total nr characters
FE = 0.1 : fraction "e" in words (0.127 in reality)

5 pt

–1 per unexplained variable

5 pt.

–1 if unnecessary digits (1 à 2 is reasonable)

–1 if not scientific notation but many 0's. (≥3 zeros)

Calculation:
NP = L/d = 3.5m / 0.1mm = 35,000 = 3.5x10^4 pages
NWP = NL * NWL = 100 * 15 = 1500 = 1.5x10^3 words per page
NW = NP * NWP = 3.5x10^4 * 1.5x10^3 = 5x10^7 words (in reality: 44M)
NC = NW * NCW = 5x10^7 * 6 = 3x10^8 characters
NE = NC * FV * FE = 3x10^8 * 0.1 = 3x10^7 "e"'s

–½ for calculation errors (each)

(see for example https://en.wikipedia.org/wiki/Wikipedia:Size_in_volumes)

NAME: ……………………………..…………………………………..……………………………
STUDENT NUMBER: s……………..…………………………………..……………………………

(spare page)